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I/IV B. Tech. DEGREE EXAMINATIONS, MARCH - 2023

First Semester

CE / AM / CS / CYS / DS / EC / EEE / ME

MATHEMATICS - I

Time : **Three Hours** Maximum : **70** Marks

Answer All Questions

Answer ONE question from each Unit. **5x14=70 M**

UNIT - I

1. a) Find whether the following system of equations are consistent if so solve them.

$x - y + 2z = 5$, $2x + y - z = 1$, $3x + y + z = 8$.

b) Show that the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ cannot be diagonalized.

(OR)

2. a) Reduce the quadratic form $10x^2 + 2y^2 + 5z^2 - 4xy - 10xz + 6yz$, find the nature of the sum of squares.

b) State Cayley-Hamilton theorem and verify the matrix, $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

UNIT - II

3. a) Verify Rolle's mean value theorem for $f(x) = e^x(\sin x - \cos x)$ in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

b) Verify Cauchy's mean value theorem for the function $f(x) = e^x$, $g(x) = e^{-x}$ in [a,b].

(OR)

4. a) Show that $|\cos b - \cos a| \leq |b - a|$ using lagranges mean value theorem.

b) Show that $\log(1+e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} + \frac{x^3}{48} + \ldots\ldots$ and hence deduce that

$\frac{e^x}{(e^x+1)} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + \ldots\ldots\ldots$ Using Maclaurins theorem.

P.T.O.

UNIT - III

5. a) Find the shortest distance from origin to the surface $xyz^2 = 2$.

   b) Define chain rule and total differential If $x = r\cos\theta$ and $y = r\sin\theta$, show that

$$\frac{\partial^2 r}{\partial x^2}+\frac{\partial^2 r}{\partial y^2}=\frac{1}{r}\left[\left(\frac{\partial r}{\partial x}\right)^2+\left(\frac{\partial r}{\partial y}\right)^2\right].$$

(OR)

6. a) If $x^x = y^y = z^z = e$, show that $\frac{\partial^2 z}{\partial x\,\partial y} = -(x\log ex)^{-1}$ if $x = y = z$.

   b) If $x + y + z = u$, $y + z = uv$, $z = uvw$, then evaluate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

UNIT - IV

7. a) By changing into polar coordinates, evaluate $\iint \frac{x^2y^2}{x^2+y^2}\,dx\,dy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ (b > a).

   b) Evaluate the integral by changing the order of the integration $\int_0^a\int_x^a \frac{e^{-y}}{y}\,dy\,dx$.

(OR)

8. a) Evaluate $\int_0^2\int_0^{\sqrt{2x-x^2}} (x^2+y^2)\,dx\,dy$ by changing to polar co-ordinates.

   b) Find the area bounded by the curves $y=x$, $y=x^2$.

UNIT - V

9. a) Find $L\left\{\int_0^t te^{-t}\sin 2t\,dt\right\}$.

   b) Find $L^{-1}\,(\cos^{-1})(s))$.

(OR)

10. a) Using Laplace transform, solve $(D^2+4D+5)y = 5$, given that $y(0)=0$, $y^1(0)= 0$.

    b) Find inverse laplace transform of $\frac{4}{(s+1)(s+2)}$.