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# I/IV B. Tech. DEGREE EXAMINATIONS, AUGUST - 2023

## First Semester

## CE / AM / CS / CYS / DS / EC / EEE / ME

## MATHEMATICS - I

Time : **Three Hours** Maximum : **70** Marks

**Answer All Questions**

**Answer ONE question from each Unit.** **5x14=70 M**

### UNIT - I

1. a) Show that only real number $\lambda$ for which the system $x+2y+3z = \lambda x$, $3x+y+2z = \lambda y$, $2x+3y+z = \lambda z$ has non-zero solution is 6 and solve them when $\lambda = 6$.

   b) Find the Eigen values and Eigen vectors of $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$.

(OR)

2. a) Diagonalizable the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

   b) State Cayley-Hamilton theorem and verify the matrix, $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix}$ and find inverse, find $A^4$.

### UNIT - II

3. a) Verify Rolle's theorem for $\log\left(\frac{x^2+ab}{x(a+b)}\right)$ in [a, b], a > 0, b > 0.

   b) Prove that $\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$.

(OR)

4. a) Verify Cauchy's mean value theorem for the function $f(x) = x^2$, $g(x) = \frac{1}{x^2}$ in [2, 4].

   b) Verify Rolle's mean value theorem for $f(x) = x(x+3)e^{\frac{-x}{2}}$ in [-3, 0].

## UNIT - III

5. a) Verify for x = u, y = u tanv, z = w, $J\left(\frac{x, y, z}{u, v, w}\right) + J^1\left(\frac{u, v, w}{x, y, z}\right) = 1$.

   b) The temperature T at any point $(x,y,z)$ in space is $T = 400\,xyz^2$, find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

(OR)

6. a) If u = f (r), where $r^2 = x^2 + y^2$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

   b) If u = log $(x^3 + y^3 + z^3 - 3xyz)$ then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.

## UNIT - IV

7. a) Find the double integration, area lying between the parabola $y = 4x - x^2$ and y = x.

   b) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2)\,dx\,dy$ by changing to polar co-ordinates.

(OR)

8. a) Evaluate $\int_0^{\pi} \int_0^{a(1+\cos\theta)} r^2 \cos\theta\, dr\, d\theta$.

   b) By changing the order of the integration evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dy\, dx$.

## UNIT - V

9. a) Using laplace transform, evaluate $\int_0^{\infty} \frac{\sin 2t}{t} dt$.

   b) Find the laplace transform of $e^{-3t}$ (cos4t + 3 sin4t).

(OR)

10. a) Find $L^{-1}\left\{\frac{S-3}{S^2+4S+13}\right\}$.

b) Using convolution theorem, find the inverse laplace transform of $\frac{1}{(s^2+9)(s+1)^2}$.